Jak řešit rovnici se substitucí, která vede k rovnici kvadratické

1. Řeš v množině reálných čísel rovnici: $2^x + \frac{16}{2^x} = 10$

<u>Řešení:</u>
substituce: $y = 2^x$

$$2^x + \frac{16}{2^x} = 10$$

$$y + \frac{16}{y} = 10 \qquad / \cdot y$$

$$y^2 + 16 = 10y$$

$$y^2 - 10y + 16 = 0$$

$$y_{1;2} = \frac{10 \pm \sqrt{100-64}}{2} = \frac{10 \pm 6}{2} \Rightarrow y_1 = 2;\ y_2 = 8$$

Zpět k substituci:

a) $2^x = 2 = 2^1 \Rightarrow x = 1$

b) $2^x = 8 = 2^3 \Rightarrow x = 3$

Zkouška:

$L(1) = 2 + 8 = 10 = P(1)$

$L(3) = 8 + 2 = 10 = P(3)$

$K = \{1;3\}$

2. Řeš v množině reálných čísel rovnici: $-2 \cdot 3^x - \frac{54}{3^x} = -56$

<u>Řešení:</u>
substituce: $y = 3^x$

$$-2y - \frac{54}{y} = -56 \qquad / \cdot y$$

$$-2y^2 - 54 = -56y \qquad / + 56y$$

$$-2y^2 + 56y - 54 = 0$$

$$-2\left(y^2 - 28y + 27\right) = 0 \qquad / : (-2)$$

$$y^2 - 28y + 27 = 0$$

$$y_{1;2} = \frac{28 \pm \sqrt{28^2 - 4 \cdot 27}}{2} = \frac{28 \pm \sqrt{784-108}}{2} = \frac{28 \pm 26}{2} \Rightarrow y_1 = 1;\ y_2 = 27$$

Zpět k substituci:

$$\text{a) } 3^x = 1 = 3^0 \Rightarrow x = 0$$

$$\text{b) } 3^x = 27 = 3^3 \Rightarrow x = 3$$

Zkouška:

$L(0) = -2 - 54 = -56 = P(0)$

$L(3) = -54 - 2 = -56 = P(3)$

$K = \{0;3\}$

3. Řeš v množině reálných čísel rovnici: $4^{2x} - 18 \cdot 4^x = -32$

Řešení:

$$4^{2x} - 18 \cdot 4^x = -32$$

$$\left(4^2\right)^x - 18 \cdot 4^x = -32$$

$$\left(4^x\right)^2 - 18 \cdot 4^x = -32$$

substituce: $y = 4^x$

$$y^2 - 18y = -32$$

$$y^2 - 18y + 32 = 0$$

$$y_{1;2} = \frac{8 \pm \sqrt{18^2 - 4 \cdot 32}}{2} = \frac{18 \pm \sqrt{196}}{2} = \frac{18 \pm 14}{2} \Rightarrow y_1 = 16;\ y_2 = 2$$

Zpět k substituci:

$$\text{a) } 4^x = 16 = 4^2 \Rightarrow x = 2$$

$$\text{b) } 4^x = 2 = \sqrt{4} = 4^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$$

Zkouška:

$L(2) = 4^{2 \cdot 2} - 18 \cdot 4^2 = 256 - 18 \cdot 16 = -32 = P(2)$

$L\left(\frac{1}{2}\right) = 4^{\frac{1}{2} \cdot 2} - 18 \cdot 4^{\frac{1}{2}} = 4 - 36 = -32 = P\left(\frac{1}{2}\right)$

$K = \left\{\frac{1}{2}; 2\right\}$

4. Řeš v množině reálných čísel rovnici: $5^x + 5^{-x+5} = 150$

Řešení:

$5^x + 5^{-x} \cdot 5^5 = 150$

$$5^x + \frac{5^5}{5^x} = 150$$

substituce: $y = 5^x$

$$y + \frac{5^5}{y} = 150 \qquad / \cdot y$$

$$y^2 + 5^5 = 150y \qquad / -150y$$

$$y^2 - 150y + 5^5 = 0$$

$$y_{1;2} = \frac{150 \pm \sqrt{150^2 - 4 \cdot 5^5}}{2} = \frac{150 \pm \sqrt{10000}}{2} = \frac{150 \pm 100}{2} \Rightarrow y_1 = 125;\ y_2 = 25$$

Zpět k substituci:

a) $5^x = 125 = 5^3 \Rightarrow x = 3$

b) $5^x = 25 = 5^2 \Rightarrow x = 2$

Zkouška:

$L(2) = 5^2 + 5^{-2+5} = 5^2 + 5^3 = 25 + 125 = 150 = P(2)$

$L(3) = 5^3 + 5^{-3+5} = 5^3 + 5^2 = 125 + 25 = 150 = P(3)$

$K = \{2; 3\}$

5. Řeš v množině reálných čísel rovnici: $2 \cdot 4^x + \frac{4}{4^x} = 9$

Řešení:

substituce: $y = 4^x$

$$2y + \frac{4}{y} = 9 \qquad / \cdot y$$

$$2y^2 + 4 = 9y \qquad / -9y$$

$$2y^2 - 9y + 4 = 0$$

$$y_{1;2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 4}}{2 \cdot 2} = \frac{9 \pm \sqrt{49}}{4} = \frac{9 \pm 7}{4} \Rightarrow y_1 = 4;\ y_2 = \frac{1}{2}$$

Zpět k substituci:

a) $4^x = 4 = 4^1 \Rightarrow x = 1$

b) $4^x = \frac{1}{2} = 2^{-1} = \left(\sqrt{4}\right)^{-1} = 4^{-\frac{1}{2}} \Rightarrow x = -\frac{1}{2}$

Zkouška:

$$L(1)=2\cdot 4^{1}+\frac{4}{4^{1}}=8+1=9=P(1)$$

$$L\left(-\frac{1}{2}\right)=2\cdot 4^{-\frac{1}{2}}+\frac{4}{4^{-\frac{1}{2}}}=2\cdot\frac{1}{2}+\frac{4}{\frac{1}{2}}=1+8=9=P\left(-\frac{1}{2}\right)$$

$$\underline{\underline{K=\left\{-\frac{1}{2};1\right\}}}$$

6. Řeš v množině reálných čísel rovnici: $3\cdot 2^{2x}+192=15\cdot 2^{x+2}$

<u>Řešení:</u>

$$3\cdot 2^{2x}+192=2^{x}\cdot 2^{2}\cdot 15$$

$$3\cdot 2^{2x}+192=60\cdot 2^{x}$$

substituce: $y=2^{x}$

$$3y^{2}+192=60y \qquad /-60y$$

$$3y^{2}-60y+192=0$$

$$3\left(y^{2}-20y+64\right)=0 \qquad /:3$$

$$y^{2}-20y+64=0$$

$$y_{1;2}=\frac{20\pm\sqrt{400-64\cdot 4}}{2}=\frac{20\pm\sqrt{144}}{2}=\frac{20\pm 12}{2}\Rightarrow y_{1}=16;\ y_{2}=4$$

Zpět k substituci:

a) $2^{x}=16=2^{4}\Rightarrow x=4$

b) $2^{x}=4=2^{2}\Rightarrow x=2$

Zkouška:

$$L(4)=3\cdot 2^{2\cdot 4}+192=960=15\cdot 64=15\cdot 2^{6}=P(4)$$

$$L(2)=3\cdot 2^{2\cdot 2}+192=240=15\cdot 16=15\cdot 2^{4}=P(2)$$

$$\underline{\underline{K=\{2;4\}}}$$

7. Řeš v množině reálných čísel rovnici: $\sin x + \frac{\sqrt{2}}{4\sin x} = \frac{\sqrt{2}+1}{2}$

Řešení:

substituce: $y = \sin x$

$$y + \frac{\sqrt{2}}{4y} = \frac{\sqrt{2}+1}{2} \qquad / \cdot y$$

$$y^2 + \frac{\sqrt{2}}{4} = \frac{\left(\sqrt{2}+1\right)y}{2} \qquad / \cdot 4$$

$$4y^2 + \sqrt{2} = 2\left(\sqrt{2}+1\right)y \qquad / -2\left(\sqrt{2}+1\right)y$$

$$4y^2 - 2\left(\sqrt{2}+1\right)y + \sqrt{2} = 0$$

$$y_{1;2} = \frac{2\left(\sqrt{2}+1\right) \pm \sqrt{4\left(\sqrt{2}+1\right)^2 - 4\cdot 4\sqrt{2}}}{8} = \frac{2\left(\sqrt{2}+1\right) \pm \sqrt{4\left(2+2\sqrt{2}+1\right) - 16\sqrt{2}}}{8} =$$

$$= \frac{2\sqrt{2}+2 \pm \sqrt{8+8\sqrt{2}+4-16\sqrt{2}}}{8} = \frac{2\sqrt{2}+2 \pm \sqrt{12-8\sqrt{2}}}{8} =$$

$$= \frac{2\sqrt{2}+2 \pm \sqrt{4\left(3-2\sqrt{2}\right)}}{8} = \frac{2\sqrt{2}+2 \pm 2\sqrt{3-2\sqrt{2}}}{8} = \frac{2\sqrt{2}+2 \pm 2\sqrt{\left(1-\sqrt{2}\right)^2}}{8} =$$

$$= \frac{2\sqrt{2}+2 \pm 2\left|\left(1-\sqrt{2}\right)\right|}{8} \Rightarrow$$

$$y_1 = \frac{2\sqrt{2}+2+\left|\left(2-2\sqrt{2}\right)\right|}{8} = \frac{2\sqrt{2}+2+\left(2\sqrt{2}-2\right)}{8} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2};$$

$$y_2 = 2\frac{2\sqrt{2}+2-\left|\left(2-2\sqrt{2}\right)\right|}{8} = \frac{2\sqrt{2}+2-\left(2\sqrt{2}-2\right)}{8} = \frac{4}{8} = \frac{1}{2}$$

Zpět k substituci:

a) $\sin x = \frac{\sqrt{2}}{2}$ b) $\sin x = \frac{1}{2}$

$x_{1k} = \frac{\pi}{4} + 2k\pi$ $x_{3k} = \frac{\pi}{6} + 2k\pi$

$x_{2k} = \frac{3}{4}\pi + 2k\pi$ $x_{4k} = \frac{5}{6}\pi + 2k\pi$

Zkouška (částečná):

$$L\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\cancel{\sqrt{2}}}{\overset{2}{\cancel{4}}\cdot\frac{\cancel{\sqrt{2}}}{\cancel{2}}} = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2} = P\left(\frac{\pi}{4}\right) \qquad L\left(\frac{\pi}{6}\right) = \frac{1}{2} + \frac{\sqrt{2}}{\overset{2}{\cancel{4}}\cdot\frac{1}{\cancel{2}}} = \frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+1}{2} = P\left(\frac{\pi}{6}\right)$$

$$K = \bigcup_{k\in Z}\left\{\frac{\pi}{6}+2k\pi; \frac{\pi}{4}+2k\pi; \frac{3}{4}\pi+2k\pi; \frac{5}{6}\pi+2k\pi\right\}$$

8. Řeš v množině reálných čísel rovnici: $\sqrt{3}\,\mathrm{tg}\,x + \frac{3}{\mathrm{tg}\,x} = 3 + \sqrt{3}$

Řešení:

substituce: $y = \mathrm{tg}\,x$

$$\sqrt{3}y + \frac{3}{y} = 3 + \sqrt{3} \qquad /\cdot y$$

$$\sqrt{3}y^2 + 3 = \left(3+\sqrt{3}\right)y \qquad /-\left(3+\sqrt{3}\right)y$$

$$\sqrt{3}y^2 - \left(3+\sqrt{3}\right)y + 3 = 0$$

$$y_{1;2} = \frac{3+\sqrt{3} \pm \sqrt{\left(3+\sqrt{3}\right)^2 - 4\cdot\sqrt{3}\cdot 3}}{2\sqrt{3}} = \frac{3+\sqrt{3} \pm \sqrt{9+6\sqrt{3}+3-12\sqrt{3}}}{2\sqrt{3}} =$$

$$= \frac{3+\sqrt{3} \pm \sqrt{9-6\sqrt{3}+3}}{2\sqrt{3}} = \frac{3+\sqrt{3} \pm \sqrt{\left(3-\sqrt{3}\right)^2}}{2\sqrt{3}} = \frac{3+\sqrt{3} \pm \left|3-\sqrt{3}\right|}{2\sqrt{3}} =$$

$$= \frac{3+\sqrt{3} \pm \left(3-\sqrt{3}\right)}{2\sqrt{3}} \Rightarrow$$

$$y_1 = \frac{3+\sqrt{3}+3-\sqrt{3}}{2\sqrt{3}} = \frac{\cancel{6}^{\,3}}{\cancel{2}\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3};$$

$$y_2 = \frac{3+\sqrt{3}-3+\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$$

Zpět k substituci:

a) $\mathrm{tg}\,x = \sqrt{3}$ a) $\mathrm{tg}\,x = 1$

$x_{1k} = \frac{\pi}{3} + k\pi$ $x_{2k} = \frac{\pi}{4} + k\pi$

Zkouška:

$$L\left(\frac{\pi}{3}\right) = \sqrt{3}\cdot\sqrt{3} + \frac{3}{\sqrt{3}} = 3 + \sqrt{3} = P\left(\frac{\pi}{3}\right)$$

$$L\left(\frac{\pi}{4}\right) = \sqrt{3}\cdot 1 + \frac{3}{1} = \sqrt{3} + 3 = 3 + \sqrt{3} = P\left(\frac{\pi}{4}\right)$$

$$K = \bigcup_{k\in Z} \left\{\frac{\pi}{4} + k\pi; \frac{\pi}{3} + k\pi\right\}$$

9. Řeš v množině reálných čísel rovnici: $2\cos^2 x - 3\cos x = -1$

<u>Řešení:</u>

substituce: $y = \cos x$

$$2y^2 - 3y = -1 \qquad /+1$$

$$2y^2 - 3y + 1 = 0$$

$$y_{1;2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} \Rightarrow y_1 = 1;\ y_2 = \frac{1}{2}$$

Zpět k substituci:

a) $\cos x = \frac{1}{2}$

$$x_{1k} = \frac{\pi}{3} + 2k\pi$$

$$x_{2k} = \frac{5}{3}\pi + 2k\pi$$

b) $\cos x = 1$

$$x_{3k} = 2k\pi$$

Zkouška:

$$L\left(\frac{\pi}{3}\right) = 2\cos^2\frac{\pi}{3} - 3\cos\frac{\pi}{3} = 2\cdot\left(\frac{1}{2}\right)^2 - 3\cdot\frac{1}{2} = \frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1 = P\left(\frac{\pi}{3}\right)$$

$$L(0) = 2\cos^2 0 - 3\cos 0 = 2\cdot 1 - 3\cdot 1 = -1 = P(0)$$

$$K = \bigcup_{k \in Z} \left\{2k\pi; \frac{\pi}{3} + 2k\pi; \frac{5}{3}\pi + 2k\pi\right\}$$

10. Řeš v množině reálných čísel rovnici: $4\sin x + \frac{2}{\sin x} = 6$

Řešení:

substituce: $y = \sin x$

$$4y + \frac{2}{y} = 6 \qquad /\cdot y$$

$$4y^2 + 2 = 6y \qquad /-6y$$

$$4y^2 - 6y + 2y = 0 \qquad /:2$$

$$2y^2 - 3y + 1 = 0$$

$$y_{1;2} = \frac{3 \pm \sqrt{9 - 4\cdot 2\cdot 1}}{4} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3\pm 1}{4} \Rightarrow y_1 = 1;\ y_2 = \frac{1}{2}$$

Zpět k substituci:

a) $\sin x = 1$

$$x_{1k} = \frac{\pi}{2} + 2k\pi$$

b) $\sin x = \frac{1}{2}$

$$x_{2k} = \frac{\pi}{6} + 2k\pi$$

$$x_{3k} = \frac{5}{6}\pi + 2k\pi$$

Zkouška:

$$L\left(\frac{\pi}{2}\right) = 4\sin\frac{\pi}{2} + \frac{2}{\sin\frac{\pi}{2}} = 4\cdot 1 + 2\cdot 1 = 6 = P\left(\frac{\pi}{2}\right)$$

$$L\left(\frac{\pi}{6}\right) = 4\sin\frac{\pi}{6} + \frac{2}{\sin\frac{\pi}{6}} = 4\cdot\frac{1}{2} + \frac{2}{\frac{1}{2}} = 2 + 4 = 6 = P\left(\frac{\pi}{6}\right)$$

$$\underline{\underline{K = \bigcup_{k\in Z}\left\{\frac{\pi}{6} + 2k\pi; \frac{\pi}{2} + 2k\pi; \frac{5}{6}\pi + 2k\pi\right\}}}$$

11. Řeš v množině reálných čísel rovnici: $4\cos x - \frac{2\sqrt{2}}{\cos x} = 2\sqrt{2} - 4$

<u>Řešení:</u>

substituce: $y = \cos x$

$$4y - \frac{2\sqrt{2}}{y} = 2\sqrt{2} - 4 \qquad / \cdot y$$

$$4y^2 - 2\sqrt{2} = \left(2\sqrt{2} - 4\right)y \qquad / - \left(2\sqrt{2} - 4\right)y$$

$$4y^2 - \left(2\sqrt{2} - 4\right)y - 2\sqrt{2} = 0 \qquad / : 2$$

$$2y^2 - \left(\sqrt{2} - 2\right)y - \sqrt{2} = 0$$

$$y_{1;2} = \frac{\sqrt{2} - 2 \pm \sqrt{\left(\sqrt{2} - 2\right)^2 + 4 \cdot 2\sqrt{2}}}{4} = \frac{\sqrt{2} - 2 \pm \sqrt{2 - 4\sqrt{2} + 4 + 8\sqrt{2}}}{4} =$$

$$= \frac{\sqrt{2} - 2 \pm \sqrt{2 + 4\sqrt{2} + 4}}{4} = \frac{\sqrt{2} - 2 \pm \sqrt{\left(\sqrt{2} + 2\right)^2}}{4} = \frac{\sqrt{2} - 2 \pm \left|\sqrt{2} + 2\right|}{4} =$$

$$= \frac{\sqrt{2} - 2 \pm \left(\sqrt{2} + 2\right)}{4} \quad \Rightarrow$$

$$y_1 = \frac{\sqrt{2} - 2 + \sqrt{2} + 2}{4} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2};$$

$$y_2 = \frac{\sqrt{2} - 2 - \sqrt{2} - 2}{4} = \frac{-4}{4} = -1$$

Zpět k substituci:

a) $\cos x = \frac{\sqrt{2}}{2}$

$$x_{1k} = \frac{\pi}{4} + 2k\pi$$

$$x_{2k} = \frac{7}{4}\pi + 2k\pi$$

b) $\cos x = -1$

$$x_{3k} = \pi + 2k\pi$$

Zkouška:

$$L\left(\frac{\pi}{4}\right) = 4 \cdot \frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = 2\sqrt{2} - 2\cancel{\sqrt{2}} \cdot \frac{2}{\cancel{\sqrt{2}}} = 2\sqrt{2} - 4 = P\left(\frac{\pi}{4}\right)$$

$$L(\pi) = 4 \cdot (-1) - \frac{2\sqrt{2}}{-1} = -4 + 2\sqrt{2} = 2\sqrt{2} - 4 = P(\pi)$$

$$K = \bigcup_{k \in Z} \left\{\frac{\pi}{4} + 2k\pi; \pi + 2k\pi; \frac{7}{4}\pi + 2k\pi\right\}$$

12. Řeš v množině reálných čísel rovnici: $6\sin x + 9 = \dfrac{-3}{\sin x}$

<u>Řešení:</u>

substituce: $y = \sin x$

$$6y + 9 = \frac{-3}{y} \qquad /\cdot y$$

$$6y^2 + 9y = -3 \qquad /+3$$

$$6y^2 + 9y + 3 = 0 \qquad /:3$$

$$2y^2 + 3y + 1 = 0$$

$$y_{1;2} = \frac{-3 \pm \sqrt{9 - 4\cdot 2}}{4} = \frac{-3 \pm \sqrt{1}}{4} \Rightarrow y_1 = -\frac{1}{2};\, y_2 = -1$$

Zpět k substituci:

a) $\sin x = -\dfrac{1}{2}$

$$x_{1k} = \frac{7}{6}\pi + 2k\pi$$

$$x_{2k} = \frac{11}{6}\pi + 2k\pi$$

b) $\sin x = -1$

$$x_{3k} = \frac{3}{2}\pi + 2k\pi$$

Zkouška:

$$L\left(\frac{7}{6}\pi\right) = 6\cdot\left(-\frac{1}{2}\right) + 9 = -3 + 9 = 6 = \frac{3}{\frac{1}{2}} = \frac{-3}{-\frac{1}{2}} = P\left(\frac{7}{6}\pi\right)$$

$$L\left(\frac{3}{2}\pi\right) = 6\cdot(-1) + 9 = 3 = \frac{-3}{-1} = P\left(\frac{3}{2}\pi\right)$$

$$K = \bigcup_{k\in Z}\left\{\frac{7}{6}\pi + 2k\pi; \frac{3}{2}\pi + 2k\pi; \frac{11}{6}\pi + 2k\pi\right\}$$

13. Řeš v množině reálných čísel rovnici: $\log x + \frac{6}{\log x} = 5$

Řešení:
substituce: $y = \log x$

$$y + \frac{6}{y} = 5 \qquad /\cdot y$$

$$y^2 + 6 = 5y \qquad /-5y$$

$$y^2 - 5y + 6 = 0$$

$$y_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 6}}{2} = \frac{5 \pm 1}{2} \quad \Rightarrow \quad y_1 = 2, \quad y_2 = 3$$

Zpět k substituci:

a) $\log x = 2 = 2\log 10 = \log 10^2 \quad \Rightarrow \quad x = 10^2 = 100$

b) $\log x = 3 = 3\log 10 = \log 10^3 \quad \Rightarrow \quad x = 10^3 = 1000$

Zkouška:

$$L(100) = 2 + \frac{6}{2} = 2 + 3 = 5 = P(100) \qquad L(1000) = 3 + \frac{6}{3} = 3 + 2 = 5 = P(1000)$$

$$K = \{100; 1000\}$$

14. Řeš v množině reálných čísel rovnici: $2\log_4^2 x + 5\log_4 x - 3 = 0$

Řešení:
substituce: $y = \log_4 x$

$$2y^2 + 5y - 3 = 0$$

$$y_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-3) \cdot 2}}{4} = \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm 7}{4} \quad \Rightarrow \quad y_1 = -3, \quad y_2 = \frac{1}{2}$$

Zpět k substituci:

a) $\log_4 x = -3 = \log_4 4^{-3} \quad \Rightarrow \quad x = 4^{-3} = \frac{1}{64}$

b) $\log_4 x = \frac{1}{2} = \log_4 4^{\frac{1}{2}} \quad \Rightarrow \quad x = 4^{\frac{1}{2}} = 2$

Zkouška:

$$L\left(\frac{1}{64}\right) = 2\log_4^2 \frac{1}{64} = 5\log_4 \frac{1}{64} - 3 = 2 \cdot 9 + 5 \cdot (-3) - 3 = 18 - 15 - 3 = 0 = P\left(\frac{1}{64}\right)$$

$$L(2) = 2\log_4^2 2 = 5\log_4 2 - 3 = 2 \cdot \frac{1}{4} + 5 \cdot \frac{1}{2} - 3 = \frac{1}{2} + \frac{5}{2} - 3 = \frac{6}{2} - 3 = 0 = P(2)$$

$$K = \left\{\frac{1}{64}; 2\right\}$$

15. Řeš v množině reálných čísel rovnici: $\log_2 x - 4 = \dfrac{5}{\log_2 x}$

Řešení:

substituce: $y = \log_2 x$

$$y - 4 = \frac{5}{y} \qquad /\cdot y$$

$$y^2 - 4y = 5 \qquad /-5$$

$$y^2 - 4y - 5 = 0$$

$$y_{1,2} = \frac{4 \pm \sqrt{16 - 4\cdot(-5)}}{2} = \frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2} \quad \Rightarrow \quad y_1 = 5, \quad y_2 = -1$$

Zpět k substituci:

a) $\log_2 x = 5 = \log_2 2^5 \quad \Rightarrow \quad x = 2^5 = 32$

b) $\log_2 x = -1 = \log_2 2^{-1} \quad \Rightarrow \quad x = 2^{-1} = \dfrac{1}{2}$

Zkouška:

$$L\left(\frac{1}{2}\right) = \log_2 32 - 4 = 5 - 4 = 1 = \frac{5}{5} = \frac{5}{\log_2 32} = P\left(\frac{1}{2}\right)$$

$$L(32) = \log_2 \frac{1}{2} - 4 = -1 - 4 = -5 = \frac{5}{-1} = \frac{5}{\log_2 \frac{1}{2}} = P(32)$$

$$\underline{\underline{K = \left\{\frac{1}{2}; 32\right\}}}$$

16. Řeš v množině reálných čísel rovnici: $\log^2 x = 1$

Řešení:

substituce: $y = \log x$

$$y^2 = 1 \qquad /-1$$

$$y^2 - 1 = 0 \quad \rightarrow \quad (y-1)(y+1) = 0 \quad \rightarrow \quad y_1 = 1, \quad y_2 = -1$$

Zpět k substituci:

a) $\log x = 1 = \log 10^1 \quad \Rightarrow \quad x = 10^1 = 10$

b) $\log x = -1 = \log 10^{-1} \quad \Rightarrow \quad x = 10^{-1} = \dfrac{1}{10}$

Zkouška:

$$L(10) = \log^2 10 = 1^2 = 1 = P(10) \qquad L\left(\frac{1}{10}\right) = \log^2 \frac{1}{10} = (-1)^2 = 1 = P\left(\frac{1}{10}\right)$$

$$\underline{\underline{K = \left\{\frac{1}{10}; 10\right\}}}$$

17. Řeš v množině reálných čísel rovnici: $\ln^2 x - 18\ln x = 40$

Řešení:

substituce: $y = \ln x$

$$y^2 - 18y = 40 \quad /-40$$

$$y^2 - 18y - 40 = 0$$

$$y_{1,2} = \frac{18 \pm \sqrt{18^2 - 4\cdot(-40)}}{2} = \frac{18 \pm \sqrt{22^2}}{2} = \frac{18 \pm 22}{2} \quad \Rightarrow \quad y_1 = 20, \quad y_2 = -2$$

Zpět k substituci:

a) $\ln x = 20 = \ln e^{20} \quad \Rightarrow \quad x = e^{20}$

b) $\ln x = -2 = \ln e^{-2} \quad \Rightarrow \quad x = e^{-2}$

Zkouška:

$$L\left(e^{20}\right) = \ln^2 e^{20} - 18\ln e^{20} = 400 - 18\cdot 20 = 40 = P\left(e^{20}\right)$$

$$L\left(e^{-2}\right) = \ln^2 e^{-2} - 18\ln e^{-2} = 4 - 18\cdot(-2) = 4 + 36 = 40 = P\left(e^{-2}\right)$$

$$\underline{\underline{K = \left\{e^{-2}; e^{20}\right\}}}$$

18. Řeš v množině reálných čísel rovnici: $\log x = \frac{8}{\log x} + 2$

Řešení:

substituce: $y = \log x$

$$y = \frac{8}{y} + 2 \quad /\cdot y$$

$$y^2 = 8 + 2y \quad /-8-2y$$

$$y^2 - 2y - 8 = 0$$

$$y_{1,2} = \frac{2 \pm \sqrt{4 - 4\cdot(-8)}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} \quad \Rightarrow \quad y_1 = 4, \quad y_2 = -2$$

Zpět k substituci:

a) $\log x = 4 = \log 10^4 \quad \Rightarrow \quad x = 10^4 = 10\,000$

b) $\log x = -2 = \log 10^{-2} \quad \Rightarrow \quad x = 10^{-2} = \frac{1}{100}$

Zkouška:

$$L(10\,000) = \log 10\,000 = 4$$

$$P(10\,000) = \frac{8}{\log 10\,000} + 2 = \frac{8}{4} + 2 = 4 \qquad L(10\,000) = P(10\,000)$$

$$L\left(\frac{1}{100}\right) = -2$$

$$P\left(\frac{1}{100}\right) = \frac{8}{-2} + 2 = -4 + 2 = -2 \qquad L\left(\frac{1}{100}\right) = P\left(\frac{1}{100}\right)$$

$$\underline{\underline{K = \left\{\frac{1}{100}; 10\,000\right\}}}$$

19. Řeš v množině reálných čísel rovnici: $30 = \log^2 x - 7\log x$

<u>Řešení:</u>

substituce: $y = \log x$

$$30 = y^2 - 7y \qquad /-30$$

$$0 = y^2 - 7y - 30$$

$$y_{1,2} = \frac{7 \pm \sqrt{7^2 + 4 \cdot 30}}{2} = \frac{7 \pm \sqrt{169}}{2} = \frac{7 \pm 13}{2} \quad \Rightarrow \quad y_1 = 10, \quad y_2 = -3$$

Zpět k substituci:

a) $\log x = 10 = \log 10^{10} \quad \Rightarrow \quad x = 10^{10}$

b) $\log x = -3 = \log 10^{-3} \quad \Rightarrow \quad x = 10^{-3} = \frac{1}{1000}$

Zkouška:

$$L\left(10^{10}\right) = 30$$

$$P\left(10^{10}\right) = \log^2 10^{10} - 7\log 10^{10} = 10^2 - 7 \cdot 10 = 100 - 70 = 30$$

$$L\left(10^{10}\right) = P\left(10^{10}\right)$$

$$L\left(\frac{1}{1000}\right) = 30$$

$$P\left(\frac{1}{1000}\right) = \log^2 \frac{1}{1000} - 7\log\frac{1}{1000} = 9 - 7 \cdot (-3) = 9 + 21 = 30$$

$$L\left(\frac{1}{1000}\right) = P\left(\frac{1}{1000}\right)$$

$$\underline{\underline{K = \left\{\frac{1}{1000}; 10^{10}\right\}}}$$

20. Řeš v množině reálných čísel rovnici: $4^x - 2\cdot 4^{\frac{x}{2}} - 8 = 0$

Řešení:

substituce: $y = 4^{\frac{x}{2}}$

$$4^{\frac{x}{2}\cdot 2} - 2\cdot 4^{\frac{x}{2}} - 8 = 0$$

$$\left(4^{\frac{x}{2}}\right)^2 - 2\cdot 4^{\frac{x}{2}} - 8 = 0$$

$$y^2 - 2y - 8 = 0$$

$$y_{1,2} = \frac{2 \pm \sqrt{4 - 4\cdot(-8)}}{2} = \frac{2 \pm 6}{2} \quad \Rightarrow \quad y_1 = 4, \quad y_2 = -2$$

Zpět k substituci:

a) $4^{\frac{x}{2}} = -2 \quad \Rightarrow$ nelze(a^x je vždy nezáporné, tedy i $4^{\frac{x}{2}}$ musí být > 0

b) $4^{\frac{x}{2}} = 4$

$$4^{\frac{x}{2}} = 4^1$$

$$\frac{x}{2} = 1 \qquad /\cdot 2$$

$$x = 2$$

Zkouška:

$$L(2) = 4^2 - 2\cdot 4^{\frac{2}{2}} - 8 = 16 - 8 - 8 = 0 = P(2)$$

$$K = \{2\}$$