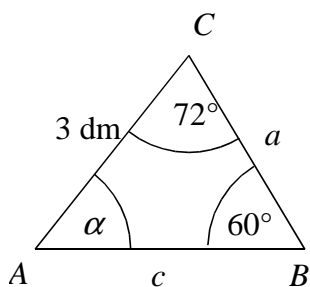


Sinová a kosinová věta

řešení

1.

a)



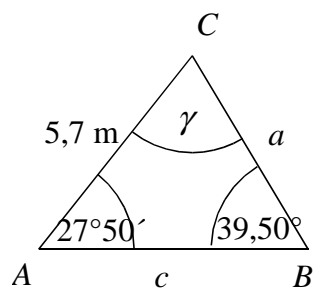
$$180^\circ = \alpha + \beta + \gamma \Rightarrow \alpha = 180^\circ - 72^\circ - 60^\circ = \underline{48^\circ}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow \frac{a}{\sin 48^\circ} = \frac{3}{\sin 60^\circ} \Rightarrow a = \frac{\sin 48^\circ}{\sin 60^\circ} \cdot 3 \doteq \underline{\underline{2,57 \text{ dm}}}$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} \Rightarrow \frac{c}{\sin 72^\circ} = \frac{3}{\sin 60^\circ} \Rightarrow c = \frac{\sin 72^\circ}{\sin 60^\circ} \cdot 3 \doteq \underline{\underline{3,29 \text{ dm}}}$$

Úhel α je 48° , délka strany a je 2,57 dm a délka strany c je 3,29 dm.

b)



$$39,5^\circ = 39^\circ 30'$$

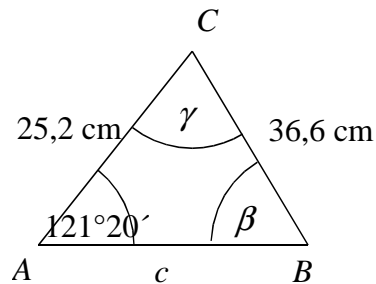
$$\gamma = 180^\circ - 27^\circ 50' - 39^\circ 30' = \underline{\underline{112^\circ 40'}}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow \frac{a}{\sin 27^\circ 50'} = \frac{5,7}{\sin 39^\circ 30'} \Rightarrow a = \frac{\sin 27^\circ 50'}{\sin 39^\circ 30'} \cdot 5,7 \doteq \underline{\underline{4,18 \text{ m}}}$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} \Rightarrow \frac{c}{\sin 112^\circ 40'} = \frac{5,7}{\sin 39^\circ 30'} \Rightarrow c = \frac{\sin 112^\circ 40'}{\sin 39^\circ 30'} \cdot 5,7 \doteq \underline{\underline{8,27 \text{ m}}}$$

Úhel γ je $112^\circ 40'$, délka strany a je 4,18 m a délka strany c je 8,27 m.

c)



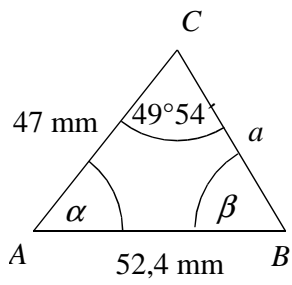
$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} \Rightarrow \frac{\sin 121^\circ 20'}{36,6} = \frac{\sin \beta}{25,2} \Rightarrow \sin \beta = \frac{25,2}{36,6} \cdot \sin 121^\circ 20' = 0,5881 \Rightarrow \underline{\underline{\beta \doteq 36^\circ}}$$

$$\gamma = 180^\circ - 36^\circ - 121^\circ 20' = \underline{\underline{22^\circ 40'}}$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} \Rightarrow c = \frac{\sin 22^\circ 40'}{\sin 36^\circ} \cdot 25,2 \doteq \underline{\underline{16,52 \text{ cm}}}$$

Úhel β je 36° , úhel γ je $22^\circ 40'$ a délka strany c je 16,52 cm.

d)



$$\frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \Rightarrow \sin \beta = \frac{47}{52,4} \cdot \sin 49^\circ 54' = 0,6861 \Rightarrow \underline{\underline{\beta \doteq 43^\circ 19'}}$$

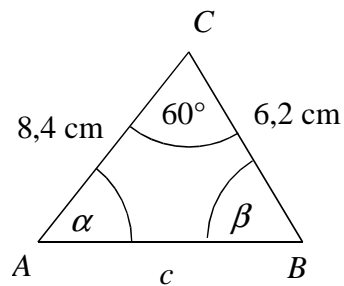
$$\alpha = 180^\circ - 49^\circ 54' - 43^\circ 19' = \underline{\underline{86^\circ 47'}}$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \Rightarrow a = \frac{\sin 86^\circ 47'}{\sin 49^\circ 54'} \cdot 52,4 \doteq \underline{\underline{68,40 \text{ mm}}}$$

Úhel β je $43^\circ 19'$, úhel α je $86^\circ 47'$ a délka strany a je $68,40$ mm.

2.

a)



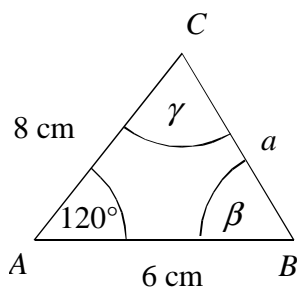
$$c^2 = a^2 + b^2 - 2ab \cos \gamma = 8,4^2 + 6,2^2 - 2 \cdot 8,4 \cdot 6,2 \cdot \cos 60^\circ = 56,92 \Rightarrow \underline{\underline{c \doteq 7,54 \text{ cm}}}$$

$$\frac{\sin \alpha}{a} = \frac{\sin \gamma}{c} \Rightarrow \sin \alpha = \frac{6,2}{7,54} \cdot \sin 60^\circ \doteq 0,71 \Rightarrow \underline{\underline{\alpha \doteq 45^\circ 24'}}$$

$$\beta = 180^\circ - 60^\circ - 45^\circ 24' = \underline{\underline{74^\circ 36'}}$$

Úhel β je $74^\circ 36'$, úhel α je $45^\circ 24'$ a délka strany c je 7,54 cm.

b)



$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

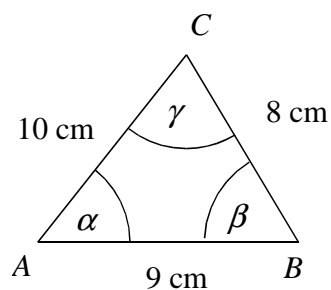
$$a^2 = 8^2 + 6^2 - 2 \cdot 8 \cdot 6 \cdot \cos 120^\circ = 0,423 \Rightarrow \underline{\underline{a \doteq 12,17 \text{ cm}}}$$

$$\frac{\sin \gamma}{c} = \frac{\sin \alpha}{a} \Rightarrow \sin \gamma = \frac{6}{12,17} \cdot \sin 120^\circ \doteq 0,423 \Rightarrow \underline{\underline{\gamma \doteq 25^\circ 17'}}$$

$$\beta = 180^\circ - 120^\circ - 25^\circ 17' = \underline{\underline{34^\circ 43'}}$$

Úhel γ je $25^\circ 17'$, úhel β je $34^\circ 43'$ a délka strany a je 12,17 cm.

c)



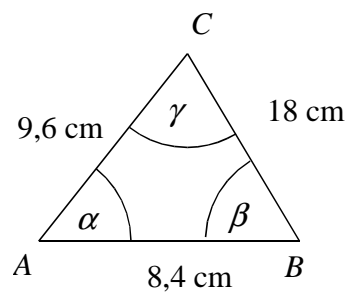
$$c^2 = a^2 + b^2 - 2ab \cos \gamma \Rightarrow \cos \gamma = \frac{c^2 - a^2 - b^2}{-2ab} = \frac{9^2 - 8^2 - 10^2}{-2 \cdot 8 \cdot 10} = 0,51875 \Rightarrow \underline{\underline{\gamma = 58^\circ 45'}}$$

$$\cos \beta = \frac{b^2 - a^2 - c^2}{-2ac} = \frac{10^2 - 8^2 - 9^2}{-2 \cdot 8 \cdot 9} = 0,3125 \Rightarrow \underline{\underline{\beta = 71^\circ 47'}}$$

$$\alpha = 180^\circ - 58^\circ 45' - 71^\circ 47' = \underline{\underline{49^\circ 28'}}$$

Úhel α je $49^\circ 28'$, úhel β je $71^\circ 47'$ a úhel γ je $58^\circ 45'$.

d)

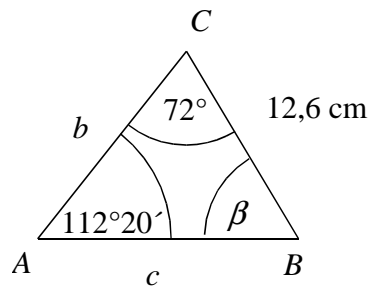


$$a = b + c \Rightarrow$$

Trojúhelníková nerovnost není splněna, trojúhelník neexistuje.

3.

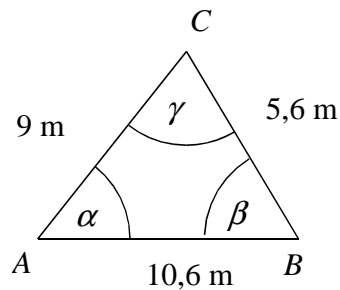
a)



$$\beta = 180^\circ - 112^\circ 20' - 72^\circ = \text{nesmysl} < 0$$

Trojúhelník neexistuje.

b)



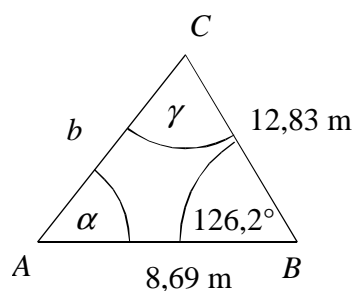
$$c^2 = a^2 + b^2 - 2ab \cos \gamma \Rightarrow \cos \gamma = \frac{c^2 - a^2 - b^2}{-2ab} = 0 \Rightarrow \underline{\underline{\gamma = 90^\circ}}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \Rightarrow \cos \alpha = \frac{a^2 - b^2 - c^2}{-2bc} = 0,8491 \Rightarrow \underline{\underline{\alpha = 31^\circ 53'}}$$

$$\beta = 180^\circ - 90^\circ - 31^\circ 53' = \underline{\underline{58^\circ 7'}}$$

Úhel γ je 90° , úhel α je $31^\circ 53'$ a úhel β je $58^\circ 7'$.

c)



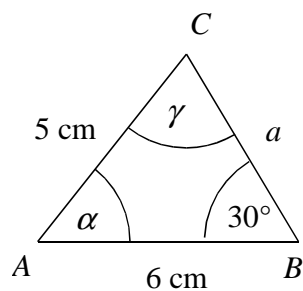
$$b^2 = a^2 + c^2 - 2ac \cos \beta = 371,82 \Rightarrow \underline{\underline{b \doteq 19,28 \text{ m}}}$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} \Rightarrow \sin \alpha = \frac{12,83}{19,28} \cdot \sin 126,2^\circ \doteq 0,537 \Rightarrow \underline{\underline{\alpha \doteq 32^\circ 29'}}$$

$$\gamma = 180^\circ - 126,2^\circ - 32,48^\circ = 21,32^\circ = \underline{\underline{21^\circ 19'}}$$

Úhel γ je $21^\circ 19'$, úhel α je $32^\circ 29'$ a délka strany b je 19,28 m.

d)



$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b} \Rightarrow \sin \gamma = \frac{6}{5} \cdot \sin 30^\circ \doteq 0,6 \Rightarrow \underline{\underline{\gamma \doteq 36^\circ 52'}}$$

$$\text{nebo } \underline{\underline{\gamma \doteq 143^\circ 08'}}$$

$$\gamma = 36^{\circ}52'$$

$$\alpha = 180^{\circ} - 36^{\circ}52' - 30^{\circ} = \underline{\underline{113^{\circ}8'}}$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \Rightarrow a = \frac{\sin 113^{\circ}8'}{\sin 36^{\circ}52'} \cdot 6 \doteq \underline{\underline{9,2 \text{ cm}}}$$

nebo:

$$\gamma = 143^{\circ}8'$$

$$\alpha = 180^{\circ} - 143^{\circ}8' - 30^{\circ} = \underline{\underline{6^{\circ}52'}}$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \Rightarrow a = \frac{\sin 6^{\circ}52'}{\sin 143^{\circ}8'} \cdot 6 \doteq \underline{\underline{1,2 \text{ cm}}}$$

Úhel γ je $36^{\circ}52'$, úhel α je $113^{\circ}8'$ a délka strany c je 9,2 cm nebo úhel γ je $143^{\circ}8'$,

úhel α je $6^{\circ}52'$ a délka strany c je 1,2 cm.